**Econometrics 3 (ECOM90013) Assignment 3**

**Let denote a simple random sample from a population with probability density function**

1. **(7 marks) Show that the sample mean is a consistent estimator of .**
2. **(1 mark) Derive a consistent method of moments estimator, say, for .**
3. **(1 mark) Specify the log-likelihood function for this sample.**
4. **(3 marks) Derive the maximum likelihood estimator, say, for and prove that it is, indeed a *maximum* likelihood estimator.**
5. **(2 marks) Derive the Fisher information for the sample.**
6. **(2 marks) Suppose that someone wishes to test the null hypothesis against the alternative that . State the true population density function and describe in words the implication for the population when this null hypothesis when this null hypothesis is true.**
7. **(12 marks) Derive the likelihood ratio, Lagrange multiplier and Wald tests for the hypotheses of Question 6. In each case provide the decision rule that you would use in practice to apply the test, including any critical value(s) you may need.**
8. **Without appeal to the generic properties of maximum likelihood estimators, prove that is consistent for .**